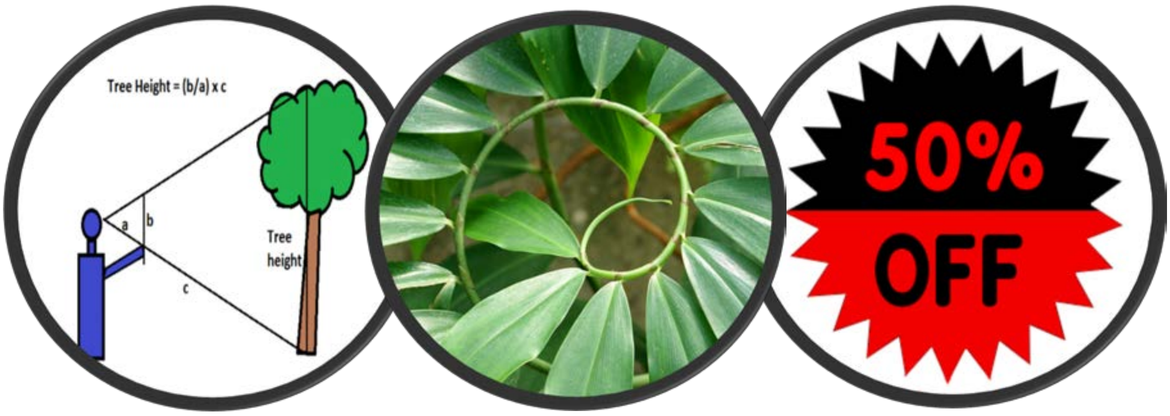
Maths Module 3

Ratio, Proportion and Percent

This module covers concepts such as:

- ratio
- direct and indirect proportion
- rates
- percent



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Module 3

Ratio, Proportion and Percent

1. Ratio
2. Direct and Indirect Proportion
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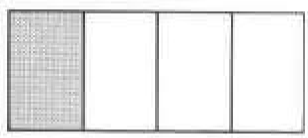
1. Ratio

Understanding ratio is very closely related to fractions. With ratio comparisons are made between equal sized parts/units. Ratios use the symbol " : " to separate the quantities being compared. For example, 1:3 means 1 unit to 3 units, and the units are the same size. Whilst ratio can be expressed as a fraction, the ratio 1:3 is NOT the same as $\frac{1}{3}$, as the rectangle below illustrates



The rectangle is 1 part grey to 3 parts white. Ratio is 1:3 (4 parts/units) The rectangle is a total of 4 parts, and therefore, the grey part represented symbolically is $\frac{1}{4}$ and not $\frac{1}{3}$

AN EXAMPLE TO BEGIN:

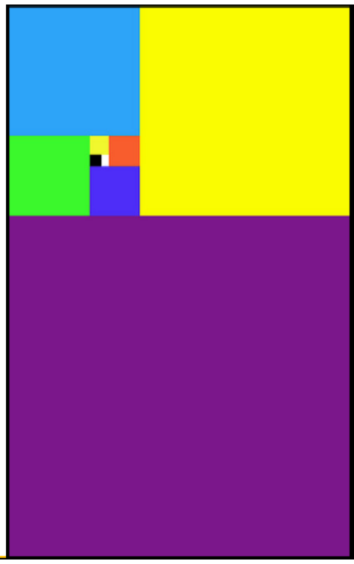
My two-stroke mower requires petrol and oil mixed to a ratio of 1:25. This means that I add one part oil to 25 parts petrol. No matter what measuring device I use, the ratio must stay the same. So if I add 200mL of oil to my tin, I add 200mL x 25 = 5000mL of petrol. Note that the total volume (oil and petrol combined) = 5200mL which can be converted to 5.2 litres. Ratio relationships are multiplicative.

Mathematically: 1:25 is the same as
2:50 is the same as
100:2500 and so on.

To verbalise we say 1 is to 25, as 2 is to 50, as 100 is to 2500, and so on……

Ratios in the real world:

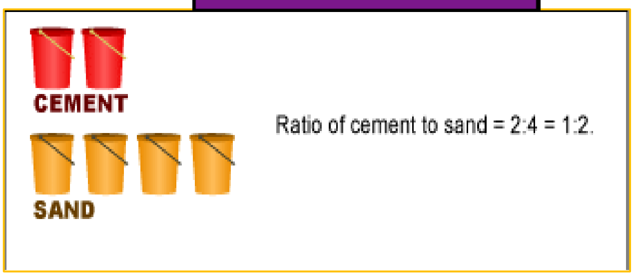
- House plans 1cm:1m = 1:100
- Map scales 1:200 000
- Circles C:D is as π: 1
- Golden ratio φ: 1 as is 1.618:1often used in art and design as shown right with the golden rectangle. This is also the rectangle ratio used for credit cards.
  (Image from http://www.flickr.com/photos/yorkjason/1456384155/)



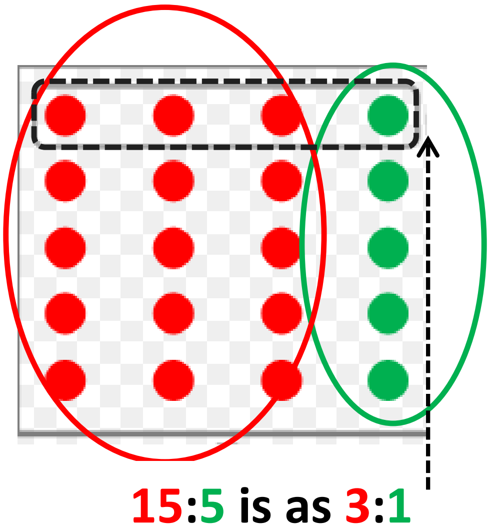
Key ideas:

Part-to-part ratio relationships

- Comparison of two quantities (e.g. number of boys to girls)
- A ratio is a way of comparing amounts
- A ratio shows the number of times an amount is contained in another, or how much bigger one amount is than another
- The two numbers are both parts of the whole
- If I need to mix some cement, then I could add two parts cement to four parts sand. Hence the ratio 2:4 (6 parts in total). The written expression is important; 4:2 would give a different mix.

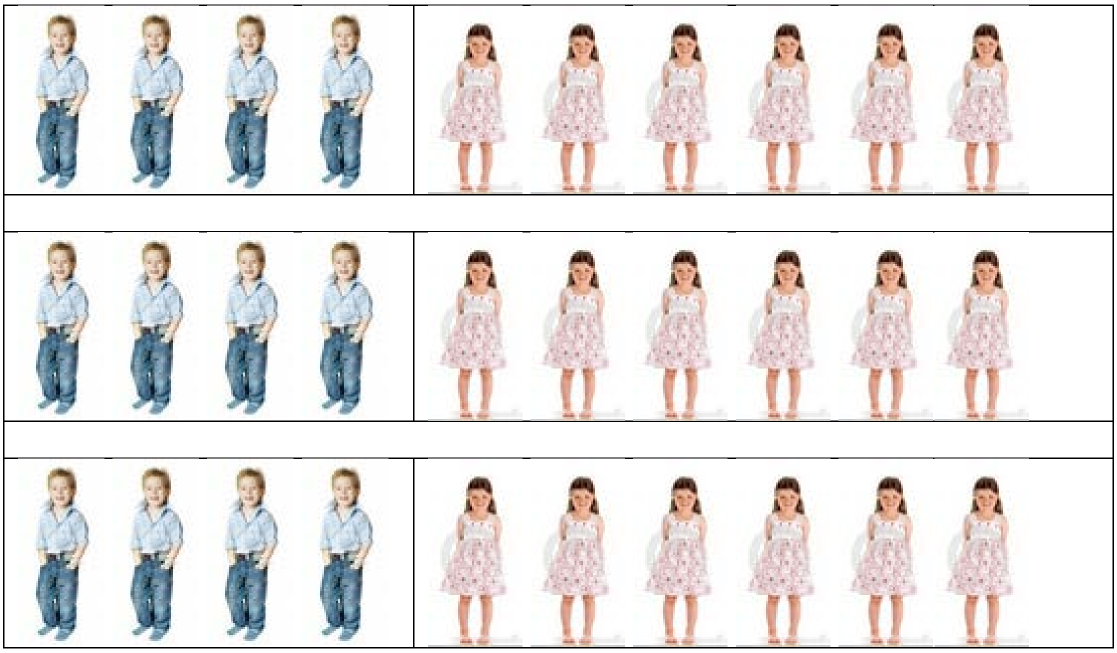


- Ratios are written to their simplest form. In the figure to the right we have 15 red dots and five green dots. The ratio is 15:5; however, this can be reduced to the simplest form as we do with fractions. The ratio as we can see in the graphic is also 3:1, if you look at the relationship of the numbers 15:5 as is 3:1, we can see that 3 is multiplied by 5 to get 15 and so is the one to get five.
- We can also think of the ratio as being a part of the whole 100%, so in the instance above, we would have 75%:25% (each being a part of 100%). If we had 100 dots, 75 of them would be red and 25 of them would be green.



Part-to-whole ratio relationship: (e.g. boys to class)

- One quantity is part of another; it is a fraction of the whole.
- For example, there are 12 boys as part of 30, which is written 12:30



1. Your Turn:

Write down as many things as you can about the ratio of the table above.

a. What is the part to part ratio?

b. What is the part to whole ratio?

c. What is the ratio of part to part in the simplest form?

d. What is the ratio of part to whole in the simplest form?

e. What is the ratio as a percentage for part to part?

2. Direct and Indirect Proportion

Indirect proportion – sometimes called 'inverse proportion'

When a variable is multiplied by a number and the other variable is divided by the same number – they are said to be in indirect proportion. For example, it takes 8 people ½ hour to mow the lawn, 4 people take one hour, 2 people 2 hours, and 1 person 4 hours.

Num. of people	**8**	**4**	**2**	**1**
Time in hours	1/2	1	2	4

Direct proportion

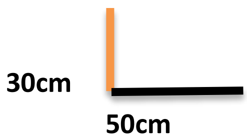
When one variable changes, the other changes in a related way; the change is constant and multiplicative. For example, if we look at the ratios, 1:25 and 2:50 we multiply both variables by two.

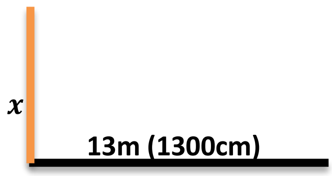
$$start\ with\ 1:25, \qquad 1 \times 2 = 2\ and\ 25 \times 2 = 50 \qquad to\ get\ 2:50.$$

Let's look at this again in a practical situation: A man plants two trees every ten minutes, four trees every 20 mins, six trees every 30 mins… 2:10, 4:20, …

We can measure the height of a tree on a sunny day without having to go up a ladder. We simply need to measure the height of a stick, in the same vicinity, and the shadows that they both cast. Here we are dealing with the cross products (meaning the multiplicative relationship) of 2 equivalent ratios.

For example, if a tree shadow is 13m, and we measure a shadow of a 30cm upright ruler in the same vicinity we can work out how tall the tree is without climbing a ladder! But first we will convert to the same unit. The stick is 30cm and its shadow is 50cm, thus 13m for the shadow of the tree is converted to 1300cm.





If we know that 30 $is\ to$ 50 is as $x\ is\ to$ 1300, then we can write $\frac{30}{50}\ is\ equivalent\ to\ \frac{x}{1300}$

Next we can use a method of cross multiplication because $x \times 50\ is\ equivalent\ to\ 30 \times 1300$

$$\frac{30}{50} \bowtie \frac{x}{1300}$$

$x \times 50 = 30 \times 1300$ we apply the rule that what we do to one side we do to the other.

$$x \times 50^{\div 50} = \frac{30 \times 1300}{50}$$

$$x = \frac{30 \times 1300^{\div 50}}{50_{\div 50}} = \frac{30 \times 26}{1}$$

$$x = 780$$

Therefore, 30:50 is the same as 780:1300 so the tree is 780cm or 7.8m tall.
Let's check 30:50 is the same as 3:5, this is also in the same proportions as 780:1300.
The change has been *constant* and *multiplicative* and one of *direct proportion.*

Without realising it, we often use ratio and direct proportion, and hence, multiplicative thinking daily. For instance, when we cook we may be required to convert measurements. We do this mentally without realising the mathematical thinking we have just so expertly applied …

Example Problem:

Making biscuits uses proportion. My recipe states that to make 50 biscuits, I need

5 cups of flour
½ a cup of sugar
1 tin of condensed milk
150g of choc chips.

How much of each ingredient will I need if I require 150 biscuits?
To calculate we need to find the ratio. If the recipe makes 50 and we need 150 we triple the recipe

(150 ÷50 = 3) 50:150 is an equivalent ratio to 1:3 that is they are in the same proportions
Therefore, 5 x 3 cups of flour = 15 cups,
$\frac{1}{2} \times \frac{3}{1} = \frac{3}{2}$ = 1 ½ cups of sugar.
$1 \times 3 =$ 3 tins of condensed milk and
150g x 3 = 450g of choc chips.

2. Your Turn:

Write each ratio in its simplest form:

a. \$450 : \$600

b. 25mm : 1.00m

c. 2.5cm : 5.00km

d. 30sec : 1hr

e. 4.5kg : 9.0g

f. 250mL : 2.00L

g. 400m : 80mm

h. 50kg : 1.20t

i. A standard white bread loaf recipe might be: 100 parts flour
65 parts warm water
2 parts salt
1.5 parts yeast

How much flour is needed if:

i. I use 50 grams of salt?
ii. I use 13 cups of water?

3. Rate

A rate is a numerical comparison between two different kinds of quantities. A rate must have units, quantity per quantity. For instance, we can buy coffee at $9.80 for every kilogram. The two variables are money and kilograms. The term 'per' is a term used in exchange for the phrase 'for every,' so we buy coffee at $9.80 per kg. Other examples:

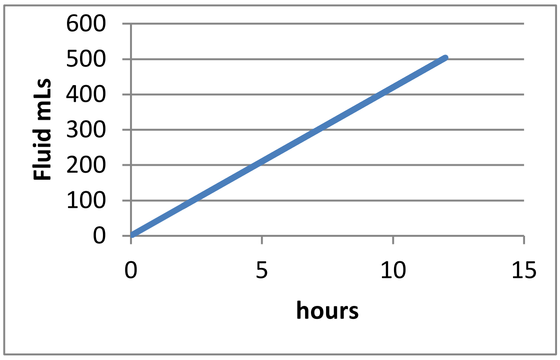
- Km per hour, km/hr
- Food prices: $ per weight
- Wages: $ per hour
- Rate can be expressed using a line graph

EXAMPLE PROBLEM 1:

An intravenous line has been inserted in a patient. Fluid is being delivered at a rate of 42mL/h. How much fluid will the patient receive in:

- 2 hours? $42\text{mL} \times 2\text{h} = 84\text{mL}$
- 8 hours? $42\text{mL} \times 8\text{h} = 336\text{mL}$
- 12 hours? $42\text{mL} \times 12\text{h} = 504\text{mL}$

A rate that is constant is related to a linear graph. The line that passes though the origin has a gradient which we call the *rise* and *run*, written as rise: run and more commonly as $rise/\ (run)$ for this graph 420/10 ∴ a positive gradient of 42. This concept will be covered more in later modules.



EXAMPLE PROBLEM 2:

William needs to drive to a neighbouring city. Part of the journey will cover 308km of winding road. He travelled 208km in two hours at an average speed over that distance. If he continues to travel at that average speed, how long will it take him to complete the 308km section of road?

Step One: Gather the facts/information

What is the distance to be covered? **308km**

What is the distance covered over what time? **112 km in 2hrs**

Step Two: Find a solution map; a formula to apply to solve the problem

1) $\frac{distance}{time} = average\ speed$

2) How to calculate the time taken: $\frac{distance\ covered}{speed} = time$

Step Three: Solve

1) Thus $\frac{112}{2} = 56$ ∴ the average speed is 56km/hr

2) Thus $\frac{308}{56} = \frac{11}{2} = 5\frac{1}{2}$

$\therefore It\ will\ take\ William\ five\ hours\ and\ thrity\ minutes\ to\ travel\ 308km.$

Step Four: Check by working backwards

$56 \times 5.5 = 308$ ✔

3. Your Turn:

Use the steps above to solve the problem below. It is good practice to apply this way of working because it will help you to structure your mathematical thinking and reasoning in the future.

a. Sonia travels a distance of 156km and then turns right to travel a further 120km. The journey takes her six hours. What is the average speed of her journey?

Step One:

Step Two:

Step Three:

Step Four:

4. Nursing Examples

Nurses will calculate IV rates: drops per minute (dpm)

Some information is required first:

- The total volume to be given, which is often written on the prescription in mLs.
- The time over which the volume is to be given, often in minutes
- The drop factor (determined by the administration set). This means how many drops per mL, which are commonly 15, 20 or 60drops/mL

$$\frac{total\ volume\ to\ be\ given\ (in\ mLs)}{time\ (in\ minutes)} \times \frac{drop\ factor}{1} = drops\ per\ minute$$

To generalise: what we are doing is dividing two variables, $\frac{x}{y}$ then we multiply that by the constant, which is the drop factor, to get k (in this case dpm); therefore, $\frac{x}{y} \times \frac{20}{1} = k$

EXAMPLE PROBLEM:

What is the IV rate for 1500mLs to be given over 10 hours with a drop factor of 20?

$$\frac{1500\text{mLs}}{10\text{hrs}} \times \frac{20}{1} = k \text{ dpm}$$

Because we are looking for drops per minute we convert the hours to minutes $(10 \times 60 = 600\text{mins})$

$$\frac{1500\text{mLs}}{600\text{mins}} \times \frac{20}{1} = 50\text{dpm}$$

In summary, this formula involves *direct proportional* thinking and reasoning. The drop factor rate is the constant, drops/mL. Hence, we are looking at the relationship of the time and the total to be given at a constant rate of 15, 20 or 60 drops per mL. Because the variables of volume and time differ and the drop factor rate remains constant, we can calculate how many drops per minute (dpm) will fall.

Drug Dosage

Another example might be calculating the dosage of a drug to be given based on an individual's weight, which involves a rate, drug per kilo. We would follow the same direct proportional thinking as above: $\frac{x}{y} \times \frac{body\ weight}{1} = k$

We would require some information to begin. It is important to always start with what we know, that is what information is on hand. For instance, what is the amount of drug $(x) \div (y)$per kilo, and then, what is the body weight?

Let's investigate; if a 60kg individual is required to take a drug that is administered at a rate of 10mg per 1kg, then we can apply the formula: $\frac{x}{y} \times \frac{}{1} = k$

Let's substitute the variables with values: $\frac{10\text{mg}}{1\text{kg}} \times \frac{60\text{kg}}{1} = k \text{ mg}$

$$\therefore \frac{10\text{mg}}{1\cancel{\text{kg}}} \times \frac{60\cancel{\text{kg}}}{1} = \frac{600\text{mg}}{1} = 600\text{mg}$$

So then what happens if the stock is given at 250mg/5mL? Let's apply the same thinking…

$\frac{x}{y} \times \frac{}{1} = k$ Now we substitute with values: $\frac{5\text{mL}}{250\text{mg}} \times \frac{600\text{mg}}{1} = k \text{ mL} \frac{5\text{mL}}{250\cancel{\text{mg}}} \times \frac{600\cancel{\text{mg}}}{1} = k \text{ mL}$

$$\frac{5\text{mL}}{250_{\div 25}} \times \frac{600^{\div 25}}{1} = \frac{5 \times 24}{10} = k \text{ mL}$$

$$\frac{5^{\div 5} \times 24}{10_{\div 5}} = \frac{24}{2} = 12 \text{ mL}$$

When you work the equations as above, you can apply the cancel out method. Always remember that what you do to one side, you do to the other. These are examples to investigate and there are many ways to work mathematically. With practise you will find your own method.

4. Your Turn:

Apply your understanding of direct proportional thinking to solve the following:

$$\frac{x}{y} \times \frac{constant}{1} = k$$

a. A patient is prescribed 150mg of soluble aspirin. We only have 300mg tablets on hand. How many tablets should be given?

b. A solution contains fluoxetine 20mg/5mL. How many milligrams of fluoxetine are in 40mL of solution?

c. A stock has the strength of 5000units per mL. What volume must be drawn up into an injection to give 6500units?

d. An intravenous line has been inserted in a patient. The total volume to be given is 1200mLs over 5hours at a drop factor of 15drops/mL. How many drops per minute will the patient receive?

e. Penicillin syrup contains 200mg of penicillin in 5mL of water. If a patient requires 300mg of penicillin how much water will be required to make the syrup?

5. Percent

Key ideas:

- The concept of percentage is an extension of the material we have covered about fractions and ratio. To allow easy comparisons of fractions we need to use the same denominator. As such, percentages use 100 as the denominator. In other words, the 'whole' is divided into 100 equal parts. The word "per cent" means per 100. Therefore 27% is $\frac{27}{100}$.

- To use percentage in a calculation, the simple mathematical procedure is modelled below where we write the percentage as a fraction with a denominator of 100. For example: 25% is $\frac{25}{100}$ thus to calculate 25%of 40, we could approach it as $\frac{25}{100} \times \frac{40}{1} = 10$

- Percentages are most commonly used to compare parts of an original. For instance, the phrase '30% off sale!' indicates that whatever the original price, the new price will be 30% less.

- However, percentages are rarely as simple as 23% of 60. Percentages are more commonly used as part of a more complex question. Often the question is, "How much is left?" or "How much was the original?"

EXAMPLE PROBLEMS:

- I am purchasing a new iPod for $300; at present, they are reduced by 15%.

  How much will my iPod cost?

Method one:

If for every $100.00 spent I save $15.00, then there are three hundreds in 300, so 15x3 is 45, [think (10x3) + (5x3)]

Step 1: SIMPLE PERCENTAGE

15% of 300 is $45.00,

Step 2: DIFFERENCE

300-45=255 therefore, the iPod will cost $255.00

Method two:

Step 1: SIMPLE PERCENTAGE

If 15% is the same as 15/100, which is also the same as 0.15, then 300 x 0.15 is 45.

Step 2: DIFFERENCE

300 – 45 is 255 therefore, the iPod will cost $255.00

Method three:

Step 1: SIMPLE PERCENTAGE

If 15% is 10% + 5%, then 10% of 300 is 30, half of 30 is 15 (5% of 300)

So $30 + 15\ is\ 45$.

Step 2: DIFFERENCE: $300 - 45 = 255$ therefore, the iPod will cost $255.00

- An advertisement at the chicken shop states that on Tuesday everything is 22% off. If chicken breasts are normally $9.99 per kilo, what is the new per kilo price?

Step 1: SIMPLE PERCENTAGE:

$$\frac{22}{100} \times \frac{9.99}{1} = 2.20$$

Step 2: DIFFERENCE: Since the price is 22% cheaper, $2.20 is subtracted from the original.

$$9.99 - 2.20 = \$7.79$$

- For the new financial year, you have been given an automatic 5% pay rise. If you were originally on $17.60 per hour, what is your new rate?

Step 1: SIMPLE PERCENTAGE: $\frac{5}{100} \times \frac{17.60}{1} = 0.88$

Step 2: DIFFERENCE: Since it is a 5% pay RISE, the $0.88 is added to the original.

17.60 + 0.88 = $18.48

Or, we could simplify this into one step:
If your new salary is the existing salary (100%) plus the increase (5%), so 105%, then you could calculate your new salary by finding 105% of the existing salary. Remembering that 105% is 1.05 as a decimal, we simply multiply 17.60 by 1.05, therefore, 17.60 x 1.05 = $18.48

- A new dress is now $237 reduced from $410. What is the percentage difference? As you can see, the problem is in reverse, so we approach it in reverse!

Step 1: DIFFERENCE

Since it is a discount the difference between the two is the discount.

$410 – $237 = $173

Step 2: PERCENTAGE: now we need to calculate what percentage of $410 was $173, and so we can use this equation: $\frac{x}{100} \times \frac{410}{1} = 173$

We can rearrange the problem in steps: $\frac{x}{100} \times \frac{410^{\div 410}}{1} = \frac{173^{\div 410}}{1}$ this step involved dividing 410 from both sides to get $\frac{x}{100} = \frac{173}{410}$

Next we work to get the x on its own, so we multiply both sides by 100. Now we have

$$x = \frac{173}{410} \times \frac{100}{1}$$

Next we solve, $(173 \times 100) \div 410 = 42.20$.

∴ The percentage difference was 42.2%.

Check: 42.2% of $410 is $173, $410 $-$ $173 = $237, the cost of the dress was $237.00.✓

- We want to buy some shoes that are reduced by 15%. The existing price is $200.00 and the new price will be 15% less. We can work this in one easy step (as above):

First, we can deduct 15% from 100% giving us 85%. Then 85% as a decimal is 0.85, so we simply use 200 x 0.85 = 170, therefore, our new price is $170.

- Now for something a bit trickier:

  What if we were to add 10%GST to an item that costs $\$5.50$.

  We can do this in one step, again converting percentage to decimals: $5.50 \times 1.10 = \$6.05$

  What if we wanted to calculate the original cost. Can we simply subtract 10% from $6.05?

  $6.05 \times 0.9 = \ \$5.45$ or $\frac{10}{100} \times \frac{6.05}{1} = 0.61\ (6.05 - 0.605 = \$5.45)$Which is close to $\$5.50$;

  but not exact, thus it is incorrect. So what do we do?

  We need to think that if 10% was added, then $6.05 is 110% so…

  To calculate we write: If $6.05 is 110%, then what was 100%?

$$\therefore \frac{6.05}{110} \times \frac{100}{1} = \frac{605}{110} = 5.5$$

Now we are back to the original cost of $5.50. ✓

- Let's try another: if we have a markup of 25% on a $300 shirt, the retail price will be $375

  $300 \times 1.25 = 375$ (using the one step method from above)

  Now let's get back to the original cost. $375 now represents 125%

  What is 100% ?

$$\frac{375}{125} \times \frac{100}{1} = \text{(divide both sides by 25)}$$

$$\frac{15^{\div 5}}{5_{\div 5}} \times \frac{100}{1} = \frac{3}{1} \times \frac{100}{1} = 300$$

5. Your Turn:

a. I want to buy some shoes that are reduced by 35%. Their original price is marked at $200. How much will I pay for the discounted shoes?

b. You have received a 6% increase on your weekly pay. If you originally received $512 per week, what is your new weekly wage?

c. In 2012, an ornithologist conducted a survey of brolgas sighted in a nearby wetland. She counted 24 individuals. In 2013, she counted 30 individuals. By what percentage did the population of brolgas increase?

d. Fiona scored 24 out of 30 on her first maths test. On her second test, she scored 22 out of 30. By what percentage did Fiona's maths results change?

e. A computer screen was $299 but is on special for $249. What is the percentage discount?

6. Combine Concepts: A Word Problem

What I need to get on my final exam?

	Grade (%)	Weight (%)
Assessment 1	30.0%	10%
Assessment 2	61.0%	15%
Assessment 3	73.2%	30%
Assessment 4	51.2%	5%
Final Exam		40%

Final Grade	Overall Percentage Needed
High Distinction	100 - 85%
Distinction	84 - 75%
Credit	74 - 65%
Pass	64 - 50%
Fail	49 - 0%

1. **How much does each of my assessments contribute to my overall percentage, which determines my final grade?**

	Calculation	Overall %
Assessment 1	This assessment contributes a maximum of 10% to my overall percentage and I scored 30.0% of those 10%, so **30.0 ÷ 100 x 10 = 3** $\frac{30}{100} \times \frac{10}{1} = 3$	3.0 %
Assessment 2	This assessment contributes a maximum of 15% to my overall percentage and I scored 61.0% of those 15%, so	
Assessment 3		
Assessment 4		
	Total	

Check: Your total should be 36.67%

2. What do I need to score on the final exam to get a P, C, or a D? Can I still get a HD?

	Calculation	Required score
P	For a Pass, I need to get at least 50% overall. I already have 36.67%, so the final exam needs to contribute **50 - 36.67 = 13.33** The exam contributes a maximum of 40% to my overall grade, but I only need to get 13.33%, so how many percent of 40% is 13.33%? **? ÷ 100 x 40 = 13.33** **? = 13.33 ÷ 40% x 100** **? = 33.33**	33.33%
C	For a Credit, I need to get at least 65% overall. I already have 36.67%, so the final exam needs to contribute: The exam contributes a maximum of 40% to my overall grade, but I only need to get , so:	
D		
HD		

Note: In most subjects, you need to score a certain percentage on the exam to pass the subject regardless of your previous results. In some cases, this may be up to 50%. Check your subject outline!

7. Answers

1) a. 12:18 b. 12:30 boys : students or 18:30 girls : students
c. 2:3 d. 2:5 boys : students or 3:5 girls : students e. 40%:60%

2) a. 3:4 b. 1:40 c. 1:200,000 d. 1:120
e. 500:1 f. 1:8 g. 5000:1 h. 1:24
i. i) 2500grams ii) 20cups

3) a. Step one: Distance covered: 156km + 120km = 276km. Time: six hours.
Step Two: Solution map: $\frac{distance\ covered}{time} = speed$
Step Three: solve: $\frac{276}{6} = 46 \ \therefore 46km/hr$
Step Four: check: $46 \times 6 = 276$

4) a. ½ tablet b. 160mg c. 1.3mL d. 60dpm e. 7.5mL

5) a. $140 b. $512 x 1.06 = $542.72 per week c. 25%

d. Score 1 = $\frac{24}{30}$ = 80%; Score 2 = $\frac{22}{30}$ = 73.3%. Fiona's score decreased (80 – 73.3) 6.7%

e. 299—249 = 50 we work out the difference first, then we put the difference over the original whole. $\frac{50}{299} \times \frac{100}{1}$ = 16.7%

6) What do I need to get on my final exam?

Credit (70.83%); Distinction (95.83%); HD (120.83 which is not achievable)

8. Helpful Websites

Ratio: http://www.mathsisfun.com/numbers/ratio.html

Direct Proportion: http://www.bbc.co.uk/skillswise/factsheet/ma19rati-l1-f-understanding-direct-proportion

Nursing Calculations: http://nursing.flinders.edu.au/students/studyaids/drugcalculations/

https://www.dlsweb.rmit.edu.au/lsu/content/c_set/nursing/nursingcalculations.html

Percentage: http://www.mathsisfun.com/percentage.html

Golden ratio: http://math2033.uark.edu/wiki/index.php/Golden_ratio